Figure 2 shows the variation of CB/CT when δ varies from 90° to 60°. It can be seen first that a strict verticality of the apparatus is useless to obtain the best performance: the ratio CB/CT is quite constant when δ decreases between 90° and 80°. Moreover, the inclination of a closed end column does not seem to improve the expected separation.

CONCLUSION

The results of this theoretical study are to be seen in an experimental perspective:

(1) The fact that the verticality of the column is not important is of practical interest in an experiment, as measurements of Soret coefficients of dilute binary solutions which can be done, using a simple method [12] with a thermogravitational column closed at both ends. In such an experiment an imperfection of the verticality of the apparatus is not prejudicial to the measurements and moreover, the apparatus is lighter and less expensive than one using a continuous flow column because it does not need a feed system to provide steady flow through the column.

(2) In the case of a flat plate column closed at both ends, the inclination of the apparatus does not seem to improve the expected separation. In the state of our results we do not know the variation of CB/CT for angles δ smaller than 60° but what we can say is that the more tilted the column is the more it approaches an horizontal thermodiffusion cell for which there is no appreciable longitudinal separation.

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EXTENSION OF THE SOLUTION OF INVERSE CONDUCTION PROBLEM

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NOMENCLATURE

b,	slab thickness;
Bi,	Biot number, hb/K;
h,	convective heat transfer coefficient;
k_0,	reference thermal conductivity at $T = T_0$;
t,	nondimensional time, $(\alpha_0\tau)/b^2$;
T,	temperature;
T_g,	driving gas temperature;
T_0,	initial temperature;
X,	dimensionless coordinate, x/b;
x,	space coordinate.

Greek symbols

α_0,	reference thermal diffusivity;
β,	constant (thermal conductivity coefficient);
τ,	time;
θ,	dimensionless temperature, $(T - T_0)/(T_g - T_0)$.

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INTRODUCTION

THE HIGH temperature range involved and the considerable variation of thermal conductivity with temperature for many present-day materials require that the variation of conductivity with temperature be considered in the analysis of the inverse conduction problem. In a previous study [1], a minimization technique was developed for the estimation of convective heat transfer coefficient and wall temperature from experimental temperature data. The current investigation represents the extension of this previously developed method by taking into consideration temperature-dependent thermal conductivity.

Beck [2] used a finite-difference approximation in conjunction with least-square fit procedure as well as nonlinear estimate method to solve the inverse conduction problem. Howard [3] developed a numerical procedure for determining the heat flux to a thermally thick wall with variable thermal properties using a single embedded thermocouple. His best results were obtained for temperature measurements close to the heated surface in con-

Table 1. Comparison of present solution with finite-difference solution of [2]

	$\theta(0, \tau)$				h, $\mathrm{W\,m^{-2}\,K^{-1}}$	
τ, s	Present	[2]	$\theta_c(1, \tau)$	$\theta_m(1, \tau)$	Present	[2]
6	0.0883	0.0838	0.0099	0.0098	536.6	581.7
7	0.1067	0.1075	0.0158	0.0159	600.6	587.0
8	0.1144	0.1116	0.0220	0.0212	592.6	598.4
9	0.1367	0.1367	0.0302	0.0302	674.2	685.3
10	0.1522	0.1545	0.0386	0.0385	712.9	693.2
11	0.1654	0.1690	0.0472	0.0472	737.4	730.0
12	0.1686	0.1639	0.0529	0.0529	718.2	721.9
13	0.1773	0.1777	0.0605	0.0605	723.6	725.8
14	0.1844	0.1813	0.0677	0.0676	723.0	725.1
15	0.1944	0.2040	0.0781	0.0782	753.6	765.0
16	0.2083	0.2174	0.0862	0.0862	758.3	770.0

junction with a small computing interval. Moreover a numerical approach needs the development of the temperature profile right from the initial state whereas an analytical approach has the advantage that temperature can be directly found at any specified location and time.

The primary objective of this note is to determine convective heat transfer coefficient and surface temperature from the measured temperature history at the outer surface of the nozzle by taking into account a linear variation of thermal conductivity with temperature.

ANALYSIS

For brevity, we shall not repeat the first four equations of [1]. But owing to temperature-dependent thermal conductivity, equation (1) of [1] is now nonlinear.

We seek to find the solution of this nonlinear conduction equation by employing iterative method [4]. Using this method, the effect of the variable thermal conductivity on the temperature distribution is taken into account by assuming that at any section X in the region $0 \leqslant X \leqslant 1$, thermal conductivity remains constant over a small interval ΔX. Considering the constant property solution as an initial guess to the foregoing nonlinear problem, an iterative scheme is set up for the temperature field for the "$i+1$" th iteration to be given by:

$$\theta^{i+1}(x, t) = 1-2\sum_{m=1}^{\infty}\frac{Bi}{(Bi^2+\lambda_m^2+Bi)}\frac{\cos\lambda_m(1-X)}{\cos\lambda_m} \times \exp[-\lambda_m^2\{1+\beta\theta^i(X,t)\}t] \quad (1a)$$

$$\lambda\tan\lambda = Bi. \quad (1b)$$

In the iterative procedure, the term on the right-hand side of equation (1a) is evaluated from the ith iteration to obtain the $i+1$ iterate directly from the left-hand side of the equation. The calculation then continues until convergence within the tolerance ε is achieved. It is important to note here that θ^i and θ^{i+1} are already known thermocouple temperatures in case of the inverse conduction problem, therefore this iteration is not needed. However it is required in the computation of temperature field other than the thermocouple's location. In the foregoing solution, Bi is an unknown parameter. In the estimation of Bi, one minimizes

$$F(Bi) = [\theta_c(X, t)-\theta_m(X, t)], \quad (2)$$

where θ_c and θ_m are, respectively, calculated and measured thermocouple temperatures.

The calculated temperature is, in general, a nonlinear function of Bi. A simple procedure approximates at each iteration the calculated temperatures by the Newton–Raphson iterative procedure (with quadratic convergence) [1]. This iterative scheme begins with an initial value of Bi and continues until $|F|$ is less than, say, 10^{-4}. The values of Bi and β for rapid convergence are 1.5708 [1] and 2.718 [4] respectively.

EXAMPLE

The iterative procedures discussed in the previous section have been utilized for estimating convective heat transfer coefficients and surface temperatures for a typical rocket nozzle divergent of mild steel in conjunction with experimentally measured outer surface temperature data [1]. The values of k_0 and β used in this investigation are 57 $\mathrm{W\,m^{-1}\,K^{-1}}$ and -1.57 [5] respectively. This inverse program, in turn, utilized this transient data to determine unknown surface conditions. Calculated results are shown in Table 1.

For the sake of comparison, the results of this analysis are compared with the finite-difference solution. The nonlinear conduction equation is solved numerically by using two-time-level, Crank–Nicholson implicit method, while a Taylor's Forward Projection method is employed to take into account the nonlinearities for achieving unconditional stability. The solution procedure is similar to that developed by Beck [2]. Twenty space intervals and a time increment of 1 s are used for computational purposes. Initial time step of 6 s is taken for starting the solution. Table 1 depicts that the results obtained by present analysis are in good agreement with the finite-difference solution.

CONCLUSIONS

The present analysis includes the temperature dependence of thermal conductivity in the solution of the inverse conduction problem. Nonlinear heat conduction equation is solved using previously developed iterative techniques. Based on known thermocouple temperature, the solution obtained by this method can be used for estimating Biot number. The results of the present analysis are compared with finite-difference solution of Beck and are found in good agreement.

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